

Another Mathematician's Apology



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AMiE President 2025 - 2026

My life and career in mathematics – education and research – could not have been more rewarding. It was a great honour to be chosen as the 1st President of AMiE, and previously as the 107th President² of the 155 year old Mathematical Association (MA) among a list comprising some giants in the world of mathematics, including: Professor G H Hardy FRS³ who worked closely with the celebrated Indian mathematician Srinivasa Ramanujan FRS⁴; Professor Sir James

Lighthill FRS⁵; Professor Sir Michael Atiyah OM FRS⁶; Professor Sir Christopher Zeeman FRS⁷; and many more besides, including another former Head of the Mathematics Department at the University of Reading, Professor E H Neville⁸.

Neville was also the MA's Librarian for over 30 years and served as the Editor of The Mathematical Gazette. At Trinity College Cambridge he was a contemporary

It is a melancholy experience for a professional mathematician to find himself writing about mathematics. The function of a mathematician is to do something, to prove new theorems, to add to mathematics, and not to talk about what he or other mathematicians have done. Statesmen despise publicists, painters despise art-critics, and physiologists, physicists, or mathematicians have usually similar feelings: there is no scorn more profound, or on the whole more justifiable, than that of the men who make for the men who explain. Exposition, criticism, appreciation, is work for second-rate minds.

Opening paragraph in G H Hardy's *A Mathematician's Apology*, 1940¹

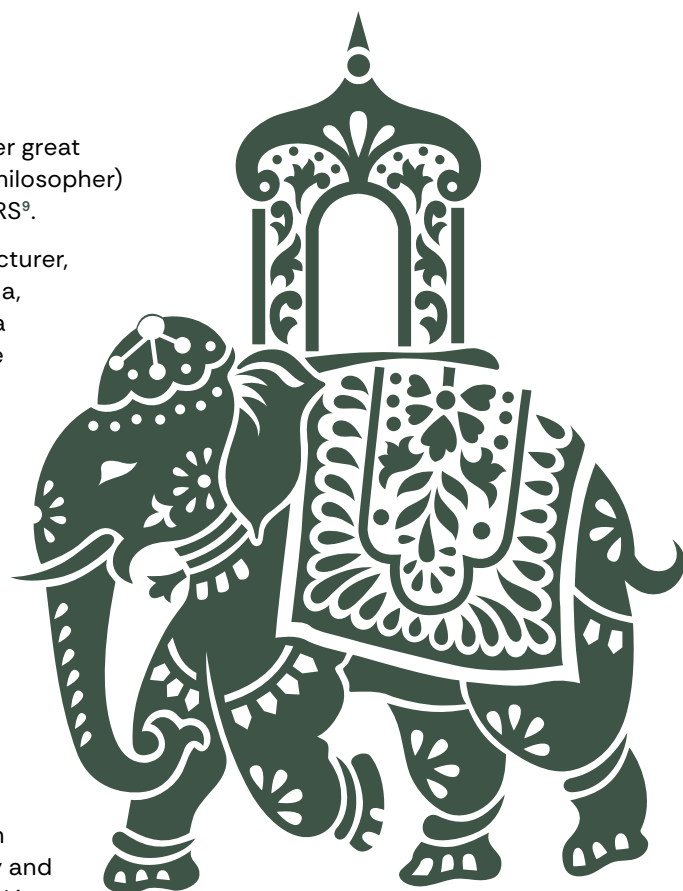
of Hardy and of another great mathematician (and philosopher) Bertrand Russell OM FRS⁹.

In 1914, as a visiting lecturer, Neville travelled to India, where, in response to a request from Hardy, he managed to persuade Ramanujan to accompany him back to England, thus playing a vital role in the initiation of one of the most celebrated mathematical collaborations of the last century^{10,11}. Most of this took place while Hardy was at Cambridge until he was appointed Savilian Professor of Geometry and of Astronomy in Oxford in 1919¹², before returning to Cambridge as Sadleirian Professor of Pure Mathematics until his retirement.

Neville worked at Reading for 35 years until his retirement and was instrumental in securing its Royal Charter as a University in 1926 (formerly University College Reading). Neville was succeeded at Reading by Professor Richard Rado FRS¹³ who was one of Hardy's doctoral students (among many other illustrious mathematicians), as was Professor Harry Pitt FRS¹⁴ who was my first Vice Chancellor at Reading. Rado's successor at Reading was Professor

John Maitland Wright FRSE whose father was Professor E M Wright FRSE¹⁵ and another of Hardy's doctoral students. We are currently celebrating our Centenary and 2026-7 will be my 50th at the University – how quickly that time has flown by!

As AMiE takes shape, the importance of the role that mathematics plays in human civilisation has never been greater, and certainly far more so than when the MA began and the aforementioned mathematicians were in their prime. Mathematics remains both an abstract intellectual pursuit



and a powerful practical tool, shaping how we interpret the world and how we act within it. It is difficult to identify any sector of modern life that is untouched by mathematics. Whether that is algorithms that govern search engines and social media platforms, the statistical models used in public health, or the optimisation techniques employed in logistics and engineering.

The very recent rapid development of artificial intelligence and machine learning provides a particularly striking example. These areas are frequently discussed in terms of innovation and disruption, but at their core lie deeply mathematical ideas: linear algebra structures underpin neural networks, probability theory governs uncertainty, and optimisation drives learning processes. Without mathematics, these technologies would simply not exist.

Similarly, the challenges facing our global society—climate change, energy sustainability, economic uncertainty, and public health—are fundamentally quantitative in nature. Mathematical models allow us to simulate complex systems, test hypotheses, and inform policy decisions. The COVID-19 pandemic offered a powerful demonstration of this capability. Epidemiological models, though imperfect, provided essential guidance for governments and health organisations. At the same time, the public scrutiny of these models revealed the importance of communicating mathematical ideas clearly and responsibly.

What emerges from these examples is not simply that mathematics is useful, but that it is indispensable. It provides a framework for reasoning about complexity, for identifying patterns, and for making informed decisions in the face of uncertainty. Moreover, it cultivates ways of thinking—precision, logical consistency, abstraction—that are valuable far beyond the discipline itself.

Education is central to meeting these challenges. Students must be equipped not only with technical skills but also with the ability to think critically about the models they use and create. They should understand the assumptions underlying mathematical methods, recognise their limitations, and appreciate the broader implications of their application.

Communication is equally important. The ability to convey mathematical ideas to non-specialists is essential in a world where decisions informed by mathematics affect everyone.

If mathematics is to fulfil its potential in the modern world, education must evolve. Traditional approaches, while providing a strong foundation, are no longer sufficient on their own. We must consider what it means to be mathematically educated in the twenty-first century and how our curricula and pedagogies can reflect that vision.

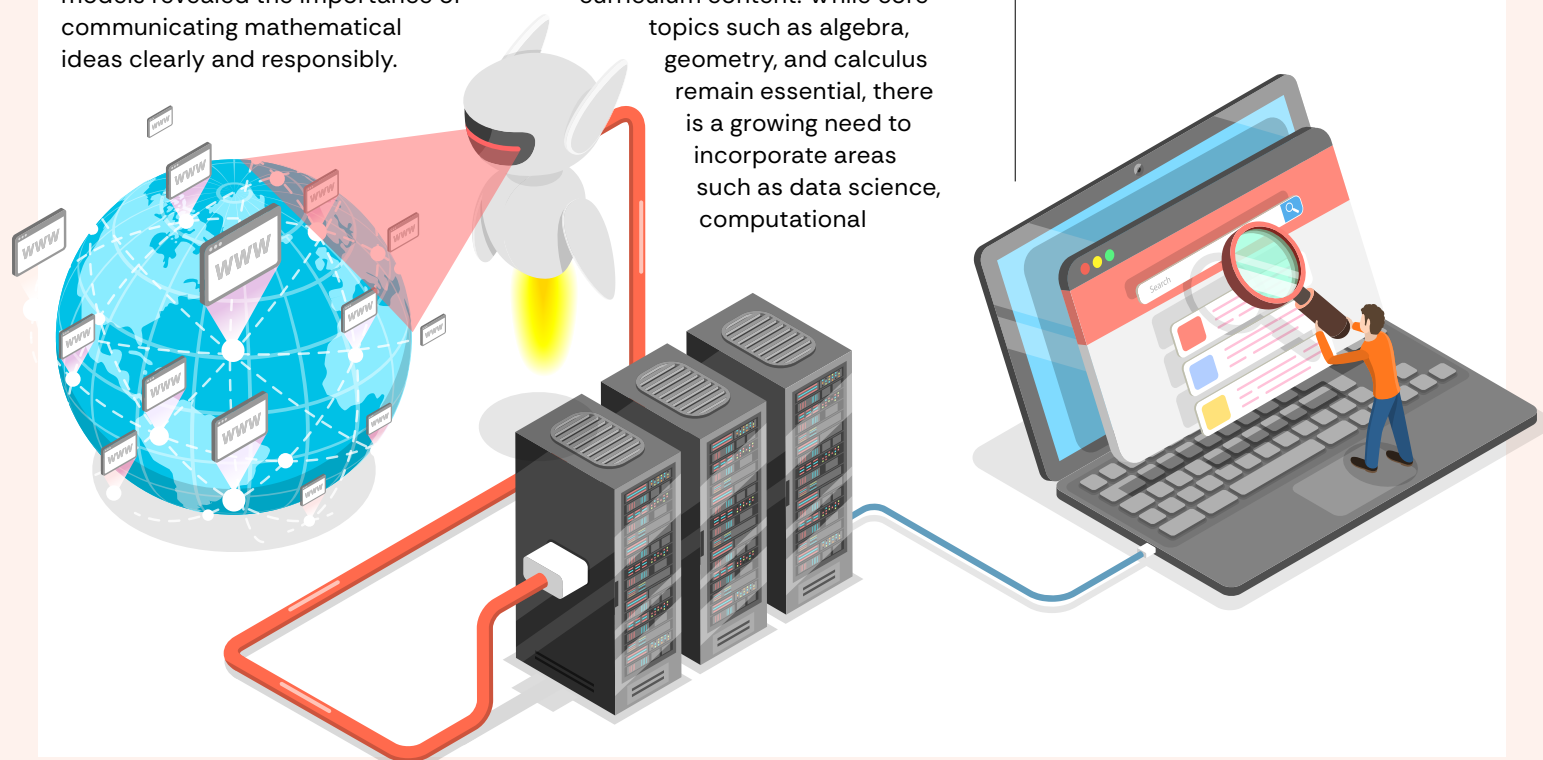
One important area of renewal is curriculum content. While core topics such as algebra, geometry, and calculus remain essential, there is a growing need to incorporate areas such as data science, computational

thinking, and mathematical modelling. These topics not only reflect contemporary applications of mathematics but also provide students with tools that are directly relevant to a wide range of careers.

Assessment practices also merit careful consideration. High-stakes examinations often emphasise procedural fluency at the expense of conceptual understanding and creative problem-solving. While fluency remains important, assessments should also value reasoning, communication, and the ability to apply mathematics in unfamiliar contexts.

A longstanding challenge in mathematics education is the perceived disconnect between what is taught in classrooms and what is required in the workplace. Employers consistently highlight the importance of skills such as problem-solving, data analysis, and communication skills that are inherently mathematical but not always explicitly developed or recognised as such.

As we look to the future, it is clear that mathematics will continue to evolve. Advances in technology, changes in the nature of work, and the emergence of new global challenges will all influence the direction of the discipline. At the same time, the fundamental nature of mathematics—as a tool for understanding, reasoning, and problem-solving—will remain unchanged.





Website

Link Building

Content

Optimization

The challenge, then, is to balance continuity with change. We must preserve the rigour and depth that characterise mathematics while embracing new ideas and approaches. We must ensure that mathematics education remains relevant and engaging, while also maintaining high standards.

As I wrote in my second report *Curriculum and assessment review and reform: evolution or revolution?*²¹⁶,

the government should look to the outstanding Royal Society report *Mathematical and Data Education (MDE)* published in September 2024¹⁷, which was the culmination of extensive research undertaken as part of their *Mathematical Futures Programme*¹⁸, to inform their priorities for mathematical and data education in the medium to long term. The report sets out several reforms necessary to develop the mass mathematical, quantitative and data skills needed for our future knowledge economy. It is deeply regrettable that the *Curriculum and Assessment Review*¹⁹, and the government response²⁰, has not heeded the advice in the MDE report.

Moving forward, AMiE has a vital role to play in addressing the challenges and opportunities outlined above. As a professional body, it provides a platform for collaboration, a source of support for educators, and a voice for the mathematical community.

We will achieve so much if we all work together in support of providing the best possible mathematics education in all settings. The Member Interest Communities (MICs) alone show the breadth of opportunities for working together as a community in support of our mission – *To foster lifelong skills, promote collaboration and make mathematics accessible, relevant, and engaging for all.*

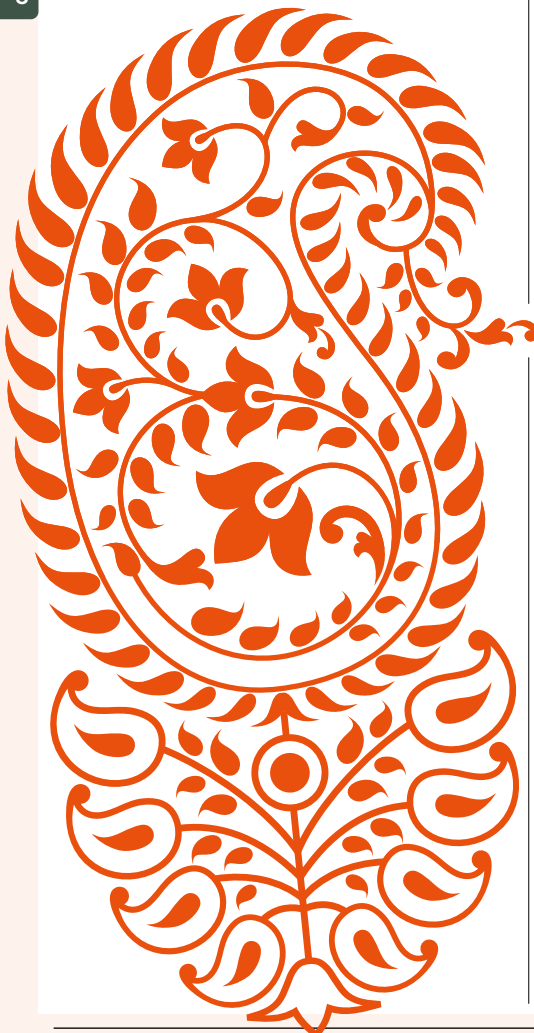
I'd now like to return full circle to the beginning of Presidency. My first article *From Coasting to Commanding – a President's mathematical journey*²¹ recounted my mathematical journey and how this has been punctuated by many lucky breaks and opportunities,

and how I have endeavoured to celebrate the beauty, power, and joy of mathematics throughout my career and while encouraging others to do the same.

My third report *Mathematics for a Desert Island*²² and my Presidential Address *Celebrating the beauty, power and joy of mathematics*²³ (scheduled to appear in the November 2026 edition of *The Mathematical Gazette*²⁴) give a pretty good indication of what makes me 'tick' mathematically and could easily form my own more modest version of Hardy's famous *A Mathematician's Apology*. I had the honour of delivering that address at the first AMiE Member Day on 11 April 2026²⁵, and what a great day that was seeing so many professional friends and colleagues, and seeing at first hand the enthusiasm, energy, ideas, and passion of members demonstrating beyond doubt that AMiE has a very promising future.

To conclude, in the spirit of Hardy's *Apology*¹ and because our terms of office as President were separated by exactly 100 years, rather than return to the many other interesting research topics and other problems in mathematics that I have been very fortunate to be able to work on, most of which have appeared in the public domain in the form of publications in journals etc²⁶, I have tried to follow in the great man's footsteps and give some of my own favourite theorems/results in mathematics:

1. *Theorem 1* – There are infinitely many primes.
2. *Theorem 2* – $\sqrt{2}$ is irrational.





Research

Social Media

Keyword

Analysis

Theorems 1 and 2 are included in the A level Mathematics Subject Content in England, and are the first two of a handful of Theorems in Hardy's *Apology* which he describes as "examples of 'real' mathematical theorems of the highest class which every mathematician will admit to be first-rate".

3. *Theorem 3* – The fundamental theorem of arithmetic: Every integer $n > 1$ can be represented uniquely as a product of prime numbers, up to the order of the factors. That is, there exist primes $p_1 < p_2 < \dots < p_k$ and positive integers n_1, n_2, \dots, n_k such that

$$n = p_1^{n_1} \cdot p_2^{n_2} \cdot \dots \cdot p_k^{n_k}$$

and this representation is unique, i.e. the primes and their multiplicities are uniquely determined by n .

The text *An Introduction to the Theory of Numbers*²⁷ by Hardy and Wright¹⁵ is a brilliant source of results in number theory.

I recall very well my introduction to differential and integral calculus and was blown away by the three key ideas:

- one can determine exactly the gradient of a given function at a point *without* using graph paper, drawing a graph, and then the tangent at the point and measuring its gradient;
- one can determine exactly the area under a graph *without* using graph paper, drawing the graph and the counting squares (approximately) under the graph;
- the two ideas in (a) and (b)

are connected through the Fundamental Theorem of Calculus:

4. *Theorem 4* – The Fundamental Theorem of Calculus, given in two forms: if f is a continuous real-valued function defined on a closed interval $[a, b]$,

- with F defined on the closed interval for $x \in [a, b]$ by $F(x) = \int_a^x f(t) dt$, then F is continuous on $[a, b]$, differentiable on the open interval on (a, b) , and $F'(x) = f(x)$ for all $x \in (a, b)$;
- if there exists a function F defined such that $F'(x) = f(x)$, then $\int_a^b f(x) dx = F(b) - F(a)$.

I still experience that sense of wonder when using or teaching calculus or its applications now just as strongly as I did all those 50+ years ago. Having spent a lifetime teaching and researching all kinds of mathematics, including modelling using differential equations and solving these numerically, I still wonder at the beauty and power of calculus and how it can be used to model the world around us and to make predictions in, say, weather and climate science, economics, health, and science, technology and engineering more generally, and in so many other fields besides.

5. *Theorem 5* – Taylor's theorem with remainder: Let f be a real-valued function that is $(n + 1)$ times continuously differentiable on an open interval containing the point a , then for any x in this interval,

$$f(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x - a)^k + R_n(x)$$

where the remainder $R_n(x)$ (in Lagrange form) is given by

$$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} (x - a)^{n+1}$$

for some $\xi \in (a, x)$.

This is an immensely powerful result for constructing and analysing numerical schemes for determining approximate solutions of differential equations, and much else in applied mathematics.

The two exceptional volumes by Richard Courant²⁸ on *Differential and Integral Calculus*^{29 30} are a rich source of theorems, results, and applications of calculus. Another text by Courant (and Robbins³¹) worth reading is the superb *What is mathematics?*³²

6. *Theorem 6* – Pappus I (centroid of a curve): If a plane curve C (not intersecting the axis of rotation) is rotated about an external axis in its plane through a full angle of 2π radians, then the surface area S of the surface of revolution generated is equal to the product of the length l of the curve and the distance travelled by the centroid of the curve, i.e. $S = 2\pi dl$, where d is the perpendicular distance from the centroid of C to the axis of rotation.
7. *Theorem 7* – Pappus II (centroid of a region): If a plane region R (not intersecting the axis of rotation) is rotated about an external axis in its plane through a full angle of 2π radians, the volume V of the solid of revolution generated equals the

product of the area A of the region and the distance travelled by the centroid of the region, i.e. $V = 2\pi dA$, where d is the perpendicular distance from the centroid of R to the axis of rotation.

I first came across Theorems 6 and 7 in the text *University Mathematics* by Blakey which I referred to in my first *Mathematical Angles* article²⁰ and discovered by chance when I was studying A level Mathematics. These are also in the Courant texts^{29,30}.

8. *Theorem 8* – e^π is irrational.

Proof, conjectures, and unresolved problems have always appealed to me, including those related to the familiar constants π and e (the base of natural logarithms). While e was shown to be irrational in 1737 (i.e. it cannot be expressed as $\frac{p}{q}$ for integers p, q , where $q \neq 0$) and that π was shown to be irrational in 1761, it wasn't until 1929 that e^π was shown to be irrational. The latter was achieved by proving that e^π is transcendental (i.e. it is not the solution of a polynomial equation with integer coefficients) and hence must also be irrational. e^π was shown to be transcendental using the Gelfond-Schneider Theorem³³ and what some mathematician's cite as the most beautiful identity in mathematics $e^{i\pi} = -1$ (named after the most prolific mathematician ever – (Leonhard) Euler³⁴), by noticing that $(-1)^{-i} = (e^{i\pi})^{-i} = e^{-i^2\pi} = e^\pi$. But which, if any, of $\pi + e, \pi e, \pi^e, \pi^\pi, e^e$ are irrational?

The teaching of formal Euclidean geometry had largely died out when I was at school, but there were four Theorems I encountered which appealed to me:

9. *Theorem 9* – Intersecting Chords: If two chords AC and BD of a circle intersect at a point P inside the circle as shown in Figure 1, then $PA \cdot PC = PB \cdot PD$, i.e. the product of the lengths of the two segments of each chord are equal.

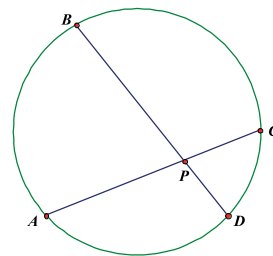


Figure 1: Intersecting chords theorem

I came across this when studying Newton's rings³⁵ and used it when carrying out associated practical experiments in A level Physics. I then learned about the connection with:

10. *Theorem 10* – Ptolemy: For a cyclic quadrilateral $ABCD$ (a quadrilateral inscribed in a circle), with sides AB, BC, CD, DA and diagonals AC, BD as shown in Figure 2, then $AC \cdot BD = AB \cdot CD + AD \cdot BC$, i.e. the product of the diagonals equals the sum of the products of opposite sides.

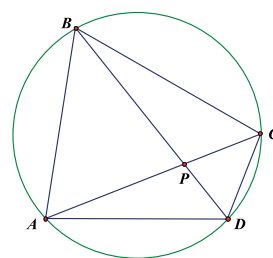


Figure 2: Ptolemy's theorem

11. *Theorem 11* – Van Aubel: For the plane quadrilateral $OABC$ shown in Figure 3 and the centres of the squares on the sides OA, AB, BC and CO denoted by P, Q, R and S , respectively, then the lines joining the mid-points of squares on opposite sides of the quadrilateral, namely PR and QS , are equal in length and perpendicular.

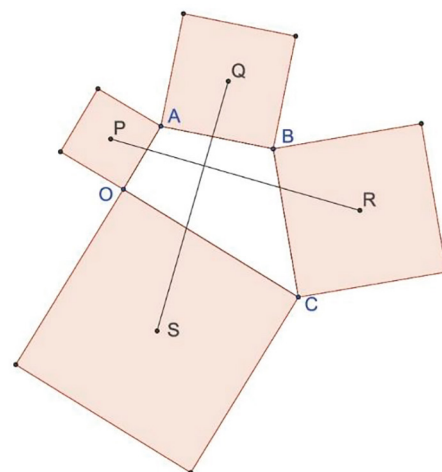
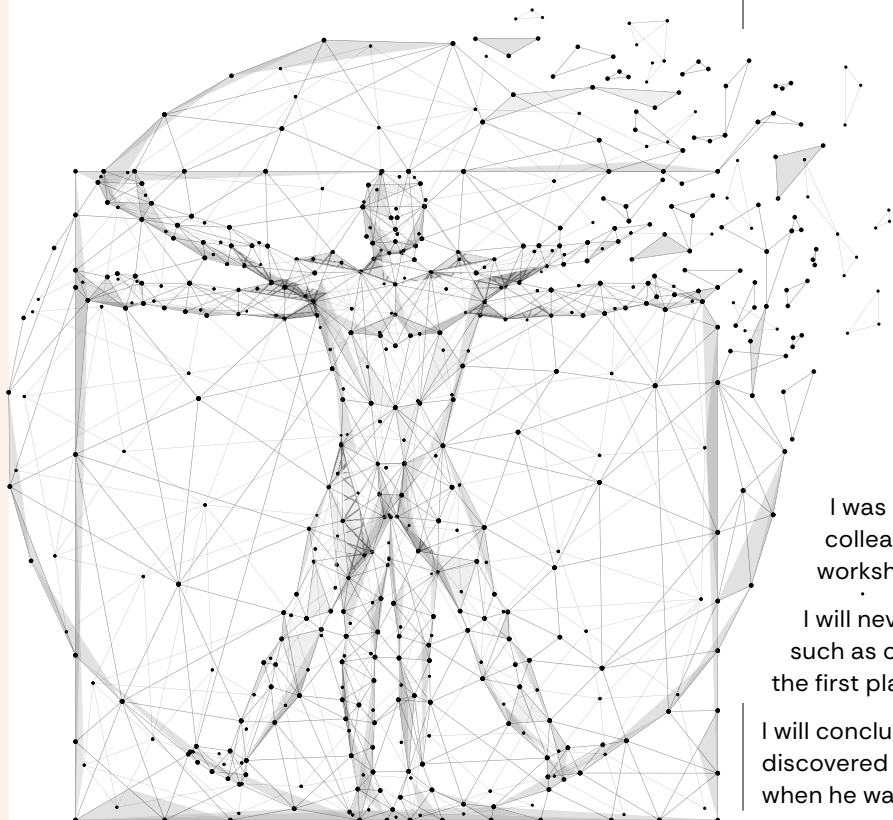


Figure 3: Van Aubel's theorem

I was introduced to this by my good friend and colleague Mick Blaylock (surprisingly at a Core Maths³⁶ workshop!).

I will never cease to be amazed by geometrical results such as ones above, and how these were discovered in the first place.

I will conclude with my absolute favourite which was discovered by the genius that was (Blaise) Pascal³⁷ in 1639 when he was a mere 16 years old:



12. *Theorem 12 – Hexagramm mysticum* (Latin for mystical hexagram): Figure 4 shows six points A, B, C, D, E and F on an ellipse and are joined by line segments AB, BC, CD, DE, EF and FA , to form a hexagram. The three pairs of opposite sides AB and DE, BC and EF, CD and FA of the hexagram, extended if necessary, and using the cyclic ordering of the vertices of $ABCDEF$, meet at three points G, H, K , respectively, which are collinear (lie on a straight line), called the Pascal line of the hexagram.

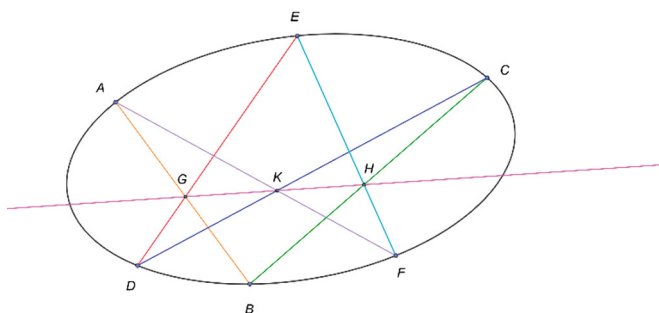


Figure 4: Pascal's theorem
(points of intersection lie inside the ellipse)

This is true for any six points, labelled in any order, for any ellipse. For example Figure 5 shows a different configuration for a different set of points for a different ellipse, where this time the hexagram $ABCDEF$ is readily seen to be a more familiar hexagon, although in this case the three pairs of opposite sides of the hexagon must be extended to meet at three points G, H, K , which in this case are external to the ellipse, but again these lie on the Pascal line of the hexagon.

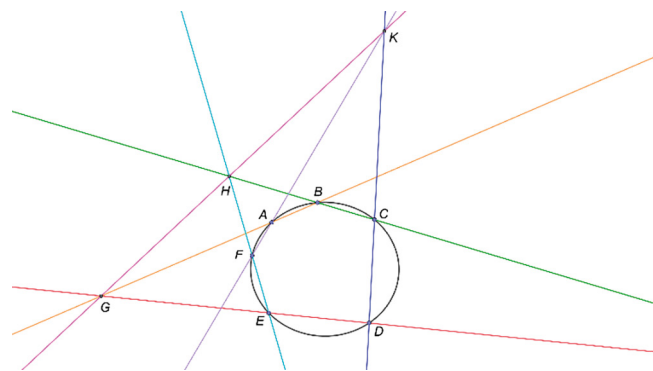


Figure 5: Pascal's theorem
(points of intersection lie outside the ellipse)

Remarkably, this is also true for any six points, labelled in any order, chosen on any of following conics – any ellipse (including any circle), any parabola or any hyperbola! Pascal wasn't fortunate to have access to the technologies we have today to explore, conjecture, and verify his proof. But you can if you wish to by downloading the GeoGebra files^{38 39} I used to create Figures 4 and 5.

All the cited texts^{27 29 30 32} are great tomes – if you have the opportunity to delve into any of these I hope you will agree that mathematics is indeed beautiful, powerful and joyous, and a discipline worthy of much celebration!

And finally (really!), a massive thank you to everyone who has supported me during my Presidency.

References

- <https://www.cambridgebookshop.co.uk/products/a-mathematicians-apology>
- or the 114th if one includes the 7 presidents of the forerunner of the MA – The Association for the Improvement of Geometrical Teaching – the first one of whom (from 1871–1878) was the celebrated Professor Thomas Hirst FRS <https://mathshistory.st-andrews.ac.uk/Biographies/Hirst/> and who had a long held belief that Euclid's Elements should be replaced as the main geometry teaching text in schools. The association soon took on board the improvement of teaching of all mathematical topics in schools and was renamed the Mathematical Association.
- <https://mathshistory.st-andrews.ac.uk/Biographies/Hardy/>
- <https://mathshistory.st-andrews.ac.uk/Biographies/Ramanujan/>
- <https://mathshistory.st-andrews.ac.uk/Biographies/Lighthill/>
- <https://mathshistory.st-andrews.ac.uk/Biographies/Atiyah/>
- <https://mathshistory.st-andrews.ac.uk/Biographies/Zeeman/>
- <https://mathshistory.st-andrews.ac.uk/Biographies/Neville/>
- <https://mathshistory.st-andrews.ac.uk/Biographies/Russell/>
- <https://www.cam.ac.uk/research/news/opinion-the-man-who-taught-infinity-how-gh-hardy-tamed-srinivasa-ramanujans-genius>
- <https://www.youtube.com/watch?v=XFsuRxoSpBU>
- <https://www.maths.ox.ac.uk/system/files/attachments/GH%20Hardy.pdf>
- https://mathshistory.st-andrews.ac.uk/Biographies/Rado_Richard/
- <https://mathshistory.st-andrews.ac.uk/Biographies/Pitt/>
- <https://mathshistory.st-andrews.ac.uk/Biographies/Wright/>
- [https://m-a.org.uk/resources/Pg_MA%20News%20October%202025%20\(3\).pdf](https://m-a.org.uk/resources/Pg_MA%20News%20October%202025%20(3).pdf)
- <https://royalsociety.org/-/media/policy/projects/math-futures/mathematical-and-data-education-policy-report.pdf>
- <https://royalsociety.org/news-resources/projects/mathematical-futures/>
- <https://www.gov.uk/government/publications/curriculum-and-assessment-review-final-report>
- <https://www.gov.uk/government/publications/curriculum-and-assessment-review-final-report-government-response>
- <https://m-a.org.uk/resources/From%20Coasting%20to%20Commanding%20-%20Paul%20Glaister.pdf>
- <https://m-a.org.uk/resources/Pg%20President%20Report%20Feb%202026.pdf>
- <https://amiememberday.org.uk/programme.asp>
- <https://www.tandfonline.com/journals/tmgz20>
- <https://amiememberday.org.uk/>
- <https://centaur.reading.ac.uk/view/creators/90000233.html>
- <https://global.oup.com/academic/product/an-introduction-to-the-theory-of-numbers-9780199219865>
- <https://mathshistory.st-andrews.ac.uk/Biographies/Courant/>
- <https://onlinelibrary.wiley.com/doi/book/10.1002/9781118033241>
- <https://onlinelibrary.wiley.com/doi/book/10.1002/9781118033234>
- <https://mathshistory.st-andrews.ac.uk/Biographies/Robbins/>
- <https://global.oup.com/academic/product/what-is-mathematics-9780195105193>
- <https://mathworld.wolfram.com/GelfondsTheorem.html>
- <https://mathshistory.st-andrews.ac.uk/Biographies/Euler>
- https://en.wikipedia.org/wiki/Newton%27s_rings
- <https://www.paulglaister.org/core-maths>
- <https://mathshistory.st-andrews.ac.uk/Biographies/Pascal/>
- [https://www.personal.reading.ac.uk/~smsglais/Figure_4_Pascal%E2%80%99s_theorem_\(points_of_intersection_lie_inside_the_ellipse\).ggb](https://www.personal.reading.ac.uk/~smsglais/Figure_4_Pascal%E2%80%99s_theorem_(points_of_intersection_lie_inside_the_ellipse).ggb)
- [https://www.personal.reading.ac.uk/~smsglais/Figure_5_Pascal%E2%80%99s_theorem_\(points_of_intersection_lie_outside_the_ellipse\).ggb](https://www.personal.reading.ac.uk/~smsglais/Figure_5_Pascal%E2%80%99s_theorem_(points_of_intersection_lie_outside_the_ellipse).ggb)